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DISCUSSION OF "NETWORK ROUTING IN A DYNAMIC ENVIRONMENT"[1]

By Andrew C. Thomas and Stephen E. Fienberg

Carnegie Mellon University

1. Introduction. An earlier version of Professor Singpurwalla's paper (which we refer to as "Singpurwalla") has served as the springboard to our own investigation of the issue of deployments of Improvised Explosive Devices, or other obstacles with a large cost to overcome, which may be placed stochastically, or by an adversarial agent, or both.

Rather than a decision-theoretic treatment, we consider a method based in part on social network analytical methods, namely, that the deployment pattern of IEDs induces a subgraph on a full road network, and that the deployment on any given road is unknown to anyone traversing the graph until arriving there, though there may be prior information on the likelihood of a deployment.

The full treatment, as acknowledged in Singpurwalla, is illustrated by Thomas and Fienberg (2011); here, we give a brief overview of our method and how it compares with Singpurwalla's approach.

2. Canadian traveler problems and network transition times. Many analyses of social networks assume that the shortest path between two individuals governs properties of their inter-relationship, and this has led to many metrics constructed using geodesic distance to approximate the importance of an individual [Freeman (1979)]. If the streets were empty of traffic, a driver on the roads will think the same way, taking the route that minimizes travel time. This is not necessarily the case when the state of the roads is uncertain, such as with traffic or construction, but is nicely encapsulated in the "Canadian Traveler Problem" formulation [Andreatta and Romeo (1988); Bar-Noy and Schieber (1991); Papadimitriou and Yannakakis (1991)]: a road may be impassable because, with some probability, there is an obstacle that cannot be traversed without waiting (in the eponymous case, a heavy snow fall). If

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the probabilities are known in advance, but the actual states of the roads are not known until reached, then an optimal route can be calculated either through exact solution or simulation, by solving for the distribution of travel times along any particular route, simulating the blockages given their propensities.

Given that all roads have some probability of a blockage, IED deployment or otherwise, we can evaluate a road's importance for travel by comparing the average travel time if the road is active to that when the road is blocked, given a source and destination. The effective difference in travel times is then a measure of the importance of the road to that travel.

Figure 1 of Singpurwalla gives three potential paths between a source A and a destination (sink) I, along which the traveler may move. If a road's state (in this case, a bridge's state) is discovered once one of its connecting intersections is reached, then this will influence the traveler as they move through the system. For this map, the traveler would know immediately whether bridge 9 was traversable; the only choice would then be if the route $CDEI$, or the route $CDEFGHI$, are shorter than the direct route CI, though the risk remains. If either of these routes is shorter when unblocked, it is the traveler's decision to try the shorter route, with the risk of having to turn back, or take the certain path without learning additional information.

3. Additional covariates. Singpurwalla's approach includes covariates in the likelihood for IED deployments on particular stretches of road in the standard fashion of a logistic regression. Examples include "local" characteristics like the proximity to a center of commerce or worship, or the nature of the road itself, such as its construction, capacity, and length, as well as circumstances particular to the timing of any particular attack, like the time of day or the weather.

For purely exploratory modeling, we can also consider the role of any road in relation to the rest of the system of roads; for example, if there are three parallel routes of equal length that a traveler can take, the likelihood that any one of these routes will be blocked will go down, all else being equal.

As we mentioned, social network analysts typically use measures of "centrality" on a graph to elicit information about the role of a node or an edge on the network, often deriving these from the role of shortest paths on the network. In this case, we can include the importance of a road in the system by considering how the road affects the Canadian Traveler: calculating the average additional travel that would be necessary if the road were closed, rather than open, when the traveler does not learn this until and unless they arrive at one of its endpoints. Because this measures the importance of a road as a conduit between two points, we have christened this quantity Canadian Betweenness Centrality; whether or not it is calculated with the possibility of other roads also being blocked is up to the analyst.

4. Expert information. Singpurwalla makes note of the encapsulation of prior information on deployments on particular road systems according to the decision maker's subjective or personal probability. Rather than including this step directly into the probability of a deployment, we recommend a slightly more roundabout approach: using the assessment of personal probabilities by the decision maker or the experts to elicit a prior distribution on the coefficients in the model [Garthwaite, Kadane and O'Hagan (2005)], here corresponding to the β terms in the logistic regression.

Essentially, the modeler queries the expert about their estimated probability of a deployment in a particular time period for all roads, then fits the model to estimate distributions on β corresponding to this uncertainty, having transformed the fractions between zero and one into an unbounded region. In the notation of Singpurwalla, we set

$$\log \frac{p_l}{1-p_l} = \sum_{i=1}^{k} Z_{il}\beta_i + \varepsilon_l,$$

where $\varepsilon_l \sim N(0, \sigma^2)$. If we define $P = [\log \frac{p_1}{1-p_1} \quad \cdots \quad \log \frac{p_n}{1-p_n}]'$, the elicited prior distribution is

$$\beta \sim N_k((Z'Z)^{-1}Z'P, (Z'Z)^{-1}\widehat{\sigma}^2).$$

If the estimate provided by the expert (or group of experts) comes from a bdistribution, we can carry out this procedure using draws from this distribution by mixing across iterations.

5. Conclusions. Singpurwalla's approach has provided a crucial stimulus to our own pursuit of the problem of road-blocking deployments. We all have a long way to go before this framework can be applied practically. As noted by Singpurwalla, the limited availability of this class of data makes it difficult to validate any of our methods, especially the adversarial nature. Even the data that are known to exist publicly, such as the Wikileaks disclosure, are not in a form that makes our frameworks applicable. We are therefore left to continue developing these ideas through simulation and thought experiment until the time comes for their more practical application.

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Department of Statistics
Carnegie Mellon University
5000 Forbes Avenue
Baker Hall 132
Pittsburgh, Pennsylvania 15213
USA
E-mail: acthomas@stat.cmu.edu
fienberg@stat.cmu.edu